DOI: https://doi.org/10.15276/aait.07.2024.7
UDC 004.05

On evaluation of reliability increase in fault-tolerant multiprocessor systems

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ABSTRACT

The work is devoted to the problem of evaluating the reliability increase of a fault-tolerant multiprocessor system by adding an extra processor to the system. It is assumed that the behavior of the modified system in the failure flow, in the case of the extra processor failure, does not differ from the behavior of the original system. The article describes both *k*-out-of-*n* systems, and more complex ones, including hierarchical systems. An important feature of the proposed approach is that it involves the preliminary calculation of some additional auxiliary values that do not depend on the reliability parameters of the added processor. Further, the reliability increase is assessed by substituting these parameter values into basic expressions, which simplifies the selection of the optimal processor from the available set, sufficient to achieve the required level of system reliability, or confirms the impossibility of this. The proposed approach is compatible with any methods of calculating the reliability parameters of fault-tolerant multiprocessor systems but is particularly relevant for methods based on statistical experiments with models of system behavior in the failure flow, in particular, such as GL-models, due to the significant computational complexity of such calculations. In addition, for the simplest cases considered, *k*-out-of-*n* systems with identical processors, a simple expression is proposed for an approximate estimate of the ratio of failure probabilities of the original and modified systems. The higher the reliability of the system processors, the higher the accuracy of such an assessment. Examples are given that prove the practical correctness of the proposed approaches. The calculation of the reliability system parameters, as well as auxiliary expressions, was based on conducting statistical experiments with corresponding GL-models.

Keywords: Fault-tolerant multiprocessor systems; incremental reliability; *k*-out-of-*n* systems; hierarchical systems; GL models

***For citation*: Romankevich, V. A., Morozov, K. V., Feseniuk, A. P., Romankevich, A. M., Zacharioudakis, L. “On evaluation of reliability increase in fault-tolerant multiprocessor systems”. *Applied Aspects of Information Technology*. 2024; Vol. 7 No. 1: 81–95. DOI: https://doi.org/10.15276/aait.07.2024.7**

INTRODUCTION

In the modern world, automation tools are increasingly being employed to manage various tasks [1, 2]. On the one hand, this allows relieving individuals from the need to perform repeatable routine tasks. On the other hand, automation overcomes human factors, such as reaction/decision-making speed, the amount of information perceived per unit of time, inattentiveness, fatigue, dependence on physical and psychological states, and so on. Moreover, in some cases, the presence of a person directly at the site of facility or the organization of remote control is either undesirable or impossible.

Management of such objects can be implemented using control systems (CS). For control systems of especially critical objects, or the so-called Critical Application or Safety-Related Systems [3, 4], [5], increased requirements for reliability are being put forward, because the failure of such systems can lead to significant material losses, threaten human health and life, etc.

In addition, since control algorithms for control systems are often quite complex and require significant computing resources, it is advisable, considering the above, to build such control

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systems on the base of fault-tolerant multiprocessor systems (FTMS) [6, 7].Fault-tolerant multiprocessor systems consists of several (sometimes dozens, hundreds, or even thousands) processors and is capable of continuing full-fledged operation if some of them fail [8].

One of the tasks faced by the developer of a fault-tolerant multiprocessor system is to assess the reliability parameters of the developed system (for example, the probability of failure-free operation) [9], particularly to verify its accordance with specified requirements. Among FTMS, there are so-called basic systems, which remain operational exactly until a certain number of (any) their processors fail. Estimating the reliability parameters of such systems is quite simple, and there are several methods available for solving this task [10, 11], [12, 13], [4, 15], [16, 17], [18, 19].

It should be noted that the structure of an FTMS is often highly diverse, primarily due to the presence of multiple different buses, sensors, and other devices. In addition, such a system may contain different processors that have different (not always compatible) instruction sets, performance levels, reliability parameters, etc. The behavior of these FTMS in the event of failures may differ from basic systems, sometimes quite significantly. Calculating the reliability parameters of such (non-basic) systems is significantly more complex, and the corresponding calculation methods are often oriented towards systems with special architectures (for example [20-48], etc.).

Universal methods for calculating the reliability parameters of such complex FTMS may, for instance, be based on conducting statistical experiments with models reflecting the system's response to the occurrence of failures. As such models, for both basic and non-basic systems, can be efficiently employed models known as GL-models [16, 17], [18], which combine the properties of graphs and Boolean functions.

As a result of the evaluation, it may be found that the reliability parameters of the developed system do not meet the specified requirements, or the requirements for a previously developed system may change towards increased reliability. In such a case, the developer needs to modify the system in a way to make it compliant with the requirements. Sometimes (especially if the required reliability is almost achieved), adding a single processor may be sufficient for this. However, the developer needs to understand what this processor should be (in terms of reliability) and where exactly in the system (e.g., in which subsystem) it should be added.

FORMULATION OF THE PROBLEM

It should be noted that the accuracy of estimating reliability parameters using the aforementioned approaches, which are based on conducting statistical experiments, depends, in particular, on the number of experiments conducted and often proves to be a resource-intensive procedure [49]. Therefore, the pursuit of an optimal system modification through direct calculation of reliability parameters for each variant in practice may be deemed excessively intricate and unfeasible.

Thus, the actual problem is to estimate the increase in reliability parameters (in particular, the probability of failure-free operation) of a system when an extra processor is added to it without the need to estimate these parameters for the modified system as a whole (this holds particular significance for non-basic systems). This work is devoted to solving this problem.

CONCEPTS AND DEFINITIONS

In the number of various fault-tolerant multiprocessor systems, it is essential to distinguish those that are resistant to failures of no more than m out of n of any of their processors, which are among the simplest, at least from the point of view of analyzing their reliability. Such systems, which are also called basic or m-out-of-n systems, will be denoted by $K(m, n)$.

Real systems, especially control systems, are not always basic. In other words, they are resistant only to some failures of a certain multiplicity, but not to other failures of the same number of processors. Such systems are called *non-basic*. These include, in particular, systems such as consecutive k-out-of-n [20-27], consecutive k-within-m-out-of-n [28, 29], [30], consecutive k-out-of-r-from-n [31, 32], m-consecutive-k-out-of-n [27, 33], [34, 35], [36] (n, f, k) [37, 38], [39], $<n, f, k>$ [27, 38], [39], consecutive-(k, l)-out-of-n [40], m-consecutive-k,l-out-of-n [41, 42], [43, 44], k_c-out-of-n [38], (r, s)-out-of-(m, n) [45, 46], [47], consecutive-k_r-out-of-n_r [48] as well as many hierarchical systems [50, 51], [52, 53].

An FTMS consists of processors, each with certain reliability parameters, such as the probability of uninterrupted operation and the probability of failure, denoted as p_i and q_i, respectively, where i is the ordinal number of the processor ($i = 1, 2, ..., n$, with n being the number of processors in the system). The probability of uninterrupted operation and failure of the system S as a whole is denoted as $P(S)$ and $Q(S)$, respectively.

BASIC SYSTEM WITH SIMILAR (I.I.D.) PROCESSORS

First, let us consider the simplest case: system S is a basic $K(m, n)$, and processors are identical. But more importantly, their reliability parameters are identical, meaning, $\forall i = 1, 2, \dots, n, p_i = p, q_i = q$. Then, $q = 1 - p$.

Let us compare the reliability of the initial system with the reliability of the system with an extra processor, assuming that the second one will also stay basic $K(m + 1, n + 1)$. The difference between these quantities in certain papers is termed *incremental reliability* [10]. We will assume that a processor identical to the processors of the original system is used, i.e. $p_{n+1} = p$, $q_{n+1} = q$. We will denote the modified system as $S \leftarrow p$.

For comparison, we will use the equation $v = \frac{1-P(S)}{1-P(S\leftarrow p)}$, i.e. the ratio of the probability of failure of the original system to the probability of failure of the modified system. To calculate the exact value of v, it is enough to calculate the values $P(S)$ and $P(S \leftarrow p)$. Recall that the original system S is basic $K(m, n)$, i.e. remains functional exactly until no more than m of n processors have failed. Let us denote $\Upsilon_{i,n}$ – the probability of exactly i of n processors failing.

It is easy to notice that $\Upsilon_{i,n} = C_n^i p^{n-i} q^i$. In this case, the probability of failure-free operation of the original system can be calculated according to the formula:

$$P(S) = \sum_{i=0}^{m} \Upsilon_{i,n} = \sum_{i=0}^{m} C_n^i p^{n-i} q^i = \\ = p^{n-m} \sum_{i=0}^{m} C_n^i p^{m-i} q^i.$$

Similarly, the modified system $S \leftarrow p$ is basic $K(m + 1, n + 1)$ and stays functional whilst no more than $m + 1$ of $n + 1$ its processors fail. Thus, the probability of its fault-free operation:

$$P(S \leftarrow p) = \sum_{i=0}^{m+1} \Upsilon_{i,n+1} = \sum_{i=0}^{m+1} C_{n+1}^i p^{n+1-i} q^i = \\ = p^{n-m} \sum_{i=0}^{m+1} C_{n+1}^i p^{m+1-i} q^i.$$

In practice, value m often is not very big ($m \ll n$), therefore, the calculation of presented expressions is a relatively simple task. However, when calculating the value v, one of the problems may be the limited accuracy of floating-point numbers. The point is that the values $P(S)$ and $P(S \leftarrow p)$ are usually very close to 1. This problem can be solved, in particular, with the use of libraries of long arithmetic and representation of p and q as fractions, i.e. ratios of integers, namely $q = \frac{a}{d}$, and $p = \frac{b}{d} = 1 - q == 1 - \frac{a}{d} = \frac{d-a}{d}$, i.e. $b = d - a$. Then

$$P = p^{n-m} \sum_{i=0}^{m} C_n^i p^{m-i} q^i = \frac{b^{n-m} \sum_{i=0}^{m} C_n^i b^{m-i} a^i}{d^n},$$
$$P' = p^{n-m} \sum_{i=0}^{m+1} C_{n+1}^i p^{m+1-i} q^i = \\ = \frac{b^{n-m} \sum_{i=0}^{m+1} C_{n+1}^i b^{m+1-i} a^i}{d^{n+1}}.$$

Thus,

$$v = \frac{1-P}{1-P'} = \frac{1 - \frac{b^{n-m} \sum_{i=0}^{m} C_n^i b^{m-i} a^i}{d^n}}{1 - \frac{b^{n-m} \sum_{i=0}^{m+1} C_{n+1}^i b^{m+1-i} a^i}{d^{n+1}}} = \\ = \frac{\frac{d^n - b^{n-m} \sum_{i=0}^{m} C_n^i b^{m-i} a^i}{d^n}}{\frac{d^{n+1} - b^{n-m} \sum_{i=0}^{m+1} C_{n+1}^i b^{m+1-i} a^i}{d^{n+1}}} = \\ = \frac{d^{n+1} - d b^{n-m} \sum_{i=0}^{m} C_n^i b^{m-i} a^i}{d^{n+1} - b^{n-m} \sum_{i=0}^{m+1} C_{n+1}^i b^{m+1-i} a^i}.$$

In one way or another, the presented formulas remain quite complex, especially for human perception. Fortunately, in the case of small values of q, which usually happens in practice, the approximate value of v can be obtained using a much simpler formula.

Theorem. For $q \to 0$ the value of the ratio $v = \frac{1-P(S)}{1-P(S\leftarrow p)} \approx \frac{m+2}{q(n+1)}$, i.e. $\lim_{q\to+0} \frac{1-P(S)}{1-P(S\leftarrow p)} = \frac{m+2}{q(n+1)}$.

Proof. Basic $K(m, n)$ system S fails if and only if more than m (out of n) arbitrary processors have failed in it, or, in other words, no more than $n - m - 1$ processors remain operational.

Thus, probability of such system to fail, $Q(S) = 1 - P(S)$ can be evaluated using next equation:

$$Q(S) = \sum_{i=0}^{n-m-1} C_n^i p^i q^{n-i} = q^n \sum_{i=0}^{n-m-1} C_n^i r^i,$$

where $r = \frac{p}{q}$.

Similarly, basic $K(m + 1, n + 1)$ system $S \leftarrow p$ fails, when more than $m + 1$ (of $n + 1$) its processors have failed, or, in other words, no more than $n - m - 1$ processors remain operational.

Thus, probability of such system to fail $Q(S \leftarrow p) = 1 - P(S \leftarrow p)$ can be evaluated using next equation:

$$Q(S \leftarrow p) = \sum_{i=0}^{n-m-1} C_{n+1}^{i} p^{i} q^{n-i+1} =$$
$$= q^{n+1} \sum_{i=0}^{n-m-1} C_{n+1}^{i} r^{i}.$$

Let us remember, that $r = \frac{p}{q} = \frac{1-q}{q} = \frac{1}{q} - 1$, i.e. when $q \to +0$ (it is obvious that the probability of failure of the processor $q \geq 0$), $r \to +\infty$. Then

$$\frac{Q(S)}{Q(S \leftarrow p)} = \frac{q^{n} \sum_{i=0}^{n-m-1} C_{n}^{i} r^{i}}{q^{n+1} \sum_{i=0}^{n-m-1} C_{n+1}^{i} r^{i}} =$$
$$= \frac{q^{n}}{q^{n+1}} \cdot \frac{\sum_{i=0}^{n-m-1} C_{n}^{i} r^{i}}{\sum_{i=0}^{n-m-1} C_{n+1}^{i} r^{i}} =$$
$$= \frac{1}{q} \cdot \frac{\sum_{i=0}^{n-m-1} C_{n}^{i} r^{i}}{\sum_{i=0}^{n-m-1} C_{n+1}^{i} r^{i}}.$$

Lets notice, that

$$\lim_{r \to +\infty} \frac{\sum_{i=0}^{n-m-1} C_{n}^{i} r^{i}}{\sum_{i=0}^{n-m-1} C_{n+1}^{i} r^{i}} = \frac{C_{n}^{n-m-1}}{C_{n+1}^{n-m-1}} =$$
$$= \frac{\dfrac{n!}{(n-m-1)! \cdot (m+1)!}}{\dfrac{(n+1)!}{(n-m-1)! \cdot (m+2)!}} =$$
$$= \frac{m+2}{n+1}.$$

Thus, when $q \to 0$ it is true that

$$\frac{\sum_{i=0}^{n-m-1} C_{n}^{i} r^{i}}{\sum_{i=0}^{n-m-1} C_{n+1}^{i} r^{i}} \approx \frac{m+2}{n+1}.$$

Meaning, $\frac{Q(S)}{Q(S \leftarrow p)} \approx \frac{1}{q} \cdot \frac{m+2}{n+1} \approx \frac{m+2}{q(n+1)}$. ■

We will also calculate the increase in the probability of a fault-free operation of the system, i.e. $\Delta P(S \leftarrow p) = P(S \leftarrow p) - P(S)$, assuming, that values ν and $P(S)$ are known. Let us remember, that

$$\nu = \frac{1-P(S)}{1-P(S \leftarrow p)}, \text{ i.e. } 1 - P(S \leftarrow p) = \frac{1-P(S)}{\nu},$$

then $P(S \leftarrow p) = \frac{P(S)-1}{\nu} + 1$.

Thus,

$$\Delta P(S \leftarrow p) = P(S \leftarrow p) - P(S) =$$
$$= \frac{P(S)-1}{\nu} + 1 - P(S) = \frac{\nu-1}{\nu} \cdot \left(1 - P(S)\right) =$$
$$= \frac{\nu-1}{\nu} \cdot Q(S) = \left(1 - \frac{1}{\nu}\right) \cdot Q(S).$$

Assuming $\nu \approx \frac{m+2}{q(n+1)}$ and substituting it into the above expression above, we get

$$\Delta P(S \leftarrow p) = \left(1 - \frac{1}{\nu}\right) \cdot Q(S) \approx$$
$$\approx \left(1 - \frac{1}{\left(\frac{m+2}{q(n+1)}\right)}\right) \cdot Q(S).$$

Simplifying this expression, we are left with

$$\Delta P(S \leftarrow p) \approx \left(1 - \frac{q(n+1)}{m+2}\right) \cdot Q(S).$$

BASIC SYSTEM WITH DIFFERENT PROCESSORS

Let us consider a more complicated case: system S is basic $K(m, n)$, however, its processors are not necessarily the same, i.e. the probabilities of their failure-free operation (or failure) may differ from each other. It is possible to calculate the probability of failure-free operation of the system $P(S)$, using any of the known methods (for example, [10, 11], [12, 13], [14, 15], [16]). Let us also review the S^{+} system containing the same processors as the original one and differing from it only in that it has 1 higher degree of fault tolerance, i.e. is basic basic $K(m + 1, n)$. For such a system, the probability of fault-free operation can be calculated in a similar way $P(S^{+})$.

Next, let us assume that an extra processor with the probability of failure-free operation was added to the system p_{n+1} and the probability of failure $q_{n+1} = 1 - p_{n+1}$. The state of this processor in the failure flow will be denoted as x_{n+1}, where 1 corresponds to its operational state, and 0 – to failure. Let us denote the resulting system as $S \leftarrow p_{n+1}$.

According to the formula of total probability:

$$\begin{aligned} P(S \leftarrow p_{n+1}) &= P\left(S \leftarrow p_{n+1}|_{x_{n+1}=0}\right) \times \\ &\times P(x_{n+1} = 0) + P\left(S \leftarrow p_{n+1}|_{x_{n+1}=1}\right) \times \\ &\times P(x_{n+1} = 1) = P\left(S \leftarrow p_{n+1}|_{x_{n+1}=0}\right) \times \\ &\times q_{n+1} + P\left(S \leftarrow p_{n+1}|_{x_{n+1}=1}\right) \cdot p_{n+1}, \end{aligned} \quad (1)$$

where $x_{n+1} = 1$ is the event meaning that the extra processor is operational, and $x_{n+1} = 0$ is not operational.

In case an extra processor fails, the behavior of such a system in the failure flow will not differ from the behavior of the original system S, i.e., $P(S \leftarrow p_{n+1}|_{x_{n+1}=0}) = P(S)$. If it is true, then its behavior in the failure flow will correspond to the behavior of the system S^{+}, because an extra processor will be able to compensate for an additional failure among the processors of the original system, i.e. $P(S \leftarrow p_{n+1}|_{x_{n+1}=1}) = P(S^+)$.

Thus:

$$P(S \leftarrow p_{n+1}) = q_{n+1}P(S) + p_{n+1}P(S^+) = \\ = (1 - p_{n+1})P(S) + p_{n+1}P(S^+) = \\ = P(S) + p_{n+1} \cdot (P(S^+) - P(S)).$$

Then,

$$\Delta P(S \leftarrow p_{n+1}) = P(S \leftarrow p_{n+1}) - P(S) = \\ = p_{n+1} \cdot (P(S^+) - P(S)).$$

Let us prove that $P(S^+) - P(S)$ equals to the probability of an event when the system S^+ is operational, and system S is not operational, i.e. $P(S^+ \cdot \bar{S})$. Remember that the system S^+ differs from the original system S having 1 higher level of fault-tolerance. Obviously $P(S^+|S) = 1$, i.e. if system S is operational, then S^+ has to be also operational.

Then by the formula of total probability

$$P(S^+) = P(S^+|S) \cdot P(S) + P(S^+|\bar{S}) \cdot P(\bar{S}) = \\ = P(S) + P(S^+|\bar{S}) \cdot P(\bar{S}).$$

Let us also note that $P(S^+|\bar{S}) \cdot P(\bar{S}) = = P(S^+ \cdot \bar{S})$. Thus,

$$P(S^+) = P(S) + P(S^+ \cdot \bar{S}).$$

Then,

$$P(S^+ \cdot \bar{S}) = P(S^+) - P(S).$$

System S^+ will be operational, and system S – will not be operational only when exactly $m + 1$ of n processors have failed in the latter. Let us denote the probability of such an event as $P(S^*)$. In some cases, calculating it may be easier than to calculating the probability $P(S^+)$. Thus, we can write:

$$\Delta P(S \leftarrow p_{n+1}) = p_{n+1} \cdot P(S^*).$$

Therefore, it is enough to calculate either both $P(S)$ and $P(S^+)$ or just $P(S^*)$ once, And then you can easily calculate the value $\Delta P(S)$ for different p_{n+1}, i.e. for various variants of the "extra" processor.

In addition, we can immediately determine the value p_{n+1}, that is sufficient to achieve the given increase $\Delta P_r(S \leftarrow p_{n+1})$ of the probability of failure-free operation of the system, namely:

$$p_{n+1} \geq \frac{P(S^*)}{\Delta P_r(S \leftarrow p_{n+1})}.$$

For the case of the system with the same components, considered earlier, takes place $P(S^*) = \Upsilon_{m+1,n} = C_n^{m+1} p^{n-m-1} q^{m+1}$. Thus,

$$\Delta P(S \leftarrow p) = P(S \leftarrow p) - P(S) = p \cdot P(S^*) = \\ = p \cdot C_n^{m+1} p^{n-m-1} q^{m+1} = C_n^{m+1} p^{n-m} q^{m+1}.$$

It should be noted that the results obtained here generally similar to the results presented in prior-arts [7, 54]. However, for non-basic systems, the situation proves to be somewhat more complex.

NON-BASIC SYSTEM

As for non-basic systems, the situation is more complicated, because the system's behavior in the failure flow won't be certain after the extra processor is connected, i.e., the system behaves differently after exposure to the same number of failures. However, looking forward we will assume that we know this behavior and it does not depend on the added processor's specifications.

In this case, we can calculate the probability of a fault-tolerant system for any given reliability parameters of added processor using any method (for example, [10, 11], [12, 13], [14, 15], [16]). Let us calculate this probability for the infinitely reliable the added processor, i.e., $p_{n+1} = 1$ and call such system as S', and the probability of its fault-free operation as $P(S')$. Also, let us point out that, in this scenario $q_{n+1} = 1 - p_{n+1} = 0$.

Equation (1) then will transform to:

$$P(S') = q_{n+1} \cdot P\left(S \leftarrow p_{n+1}|_{x_{n+1}=0}\right) + \\ + p_{n+1} \cdot P\left(S \leftarrow p_{n+1}|_{x_{n+1}=1}\right) = \\ = 0 \cdot P\left(S \leftarrow p_{n+1}|_{x_{n+1}=0}\right) + 1 \times \\ \times P\left(S \leftarrow p_{n+1}|_{x_{n+1}=1}\right) = P\left(S \leftarrow p_{n+1}|_{x_{n+1}=1}\right).$$

Also let us assume that, in case of an extra processor failure, the system won't differ from original from the failure flow's point of view. Therefore, it will be fair

$$P\left(S \leftarrow p_{n+1}|_{x_{n+1}=0}\right) = P(S).$$

Next, let us rewrite the equation (1) again, this time for arbitrary value p_{n+1} and $q_{n+1} = 1 - p_{n+1}$:

$$P(S \leftarrow p_{n+1}) = q_{n+1} \cdot P\left(S \leftarrow p_{n+1}|_{x_{n+1}=0}\right) + \\ + p_{n+1} \cdot P\left(S \leftarrow p_{n+1}|_{x_{n+1}=1}\right) = \\ = q_{n+1} \cdot P(S) + p_{n+1} \cdot P(S').$$

Then, we can calculate the increase in the probability of fault-free system operation:

$$\Delta P(S \leftarrow p_{n+1}) = P(S \leftarrow p_{n+1}) - P(S) = \\ = q_{n+1} \cdot P(S) + p_{n+1} \cdot P(S') - P(S) = \\ = p_{n+1} \cdot P(S') - (1 - q_{n+1}) \cdot P(S) = \\ = p_{n+1} \cdot \left(P(S') - P(S)\right).$$

Designating $\Delta P(S') = P(S') - P(S)$, we get

$$\Delta P(S \leftarrow p_{n+1}) = p_{n+1} \cdot \Delta P(S').$$

Thus, it turns out that it is enough to calculate once, in one way or another, the values of the probabilities of failure-free operation of both the original system S and the system with an extra infinitely reliable processor, after that the increase of reliability due to the addition of an extra processor can be easily assessed.

We can also formulate the equation for assessing the value p_{n+1}, which is enough to achieve the required value of increase $\Delta P_r(S \leftarrow p_{n+1})$ of probability of fault-free system operation

$$p_{n+1} \geq \frac{P(S') - P(S)}{\Delta P_r(S \leftarrow p_{n+1})},$$

or, if the mentioned above notation $\Delta P(S')$ is used, we get

$$p_{n+1} \geq \frac{\Delta P(S')}{\Delta P_r(S \leftarrow p_{n+1})}.$$

It is also important to note that, in the case of basic systems, $\Delta P(S')$ precisely corresponds to $P(S^*)$. Indeed, let us remember that $P(S^*) = P(S^+) - P(S)$, where $P(S^+)$ is the probability of fault-free operation of the system with degree of fault-tolerance that is 1 higher, than in the original system has. It is easy to notice, that this probability will be equal to probability $P(S')$ of fault-free operation of the considered here system with an extra, infinitely reliable, processor. Thus, the formula proposed in this section fits every system type, mentioned above, and is the most general. However, the use of particular formulae, proposed in previous sections, can be preferable in terms of computational complexity.

HIERARCHICAL SYSTEM WITH NON-BASIC SUBSYSTEMS AND DIFFERENT PROCESSORS

The addition of a processor to a hierarchical system deserves special consideration. Let the system S contain k subsystems s_j. Where $1 \leq j \leq k$. And let n be the total number of the system's processors. Let us assume, that the developer of the system decided to increase its reliability by adding a processor with probability of fault-free operation p_{n+1}. The increase in probability of fault-free operation for such system $\Delta P(S)$ will not only depend on p_{n+1} but also depends on which subsystem the extra processor will be added to. Naturally arises, the task of selecting such a subsystem to obtain the maximum increase in reliability.

Let us remember, that assessing the probability of fault-free operation of a hierarchical system is quite a resource-intensive task, the solution to which in the reasonable time is to carry out step-by-step statistical experiments with GL-models for each subsystem separately and then for the top-level system [52, 53].

Let us designate via $s_j \leftarrow p_{n+1}$ the subsystem, that we get by adding a processor to the subsystem s_j, and through $S \leftarrow s_j \leftarrow p_{n+1}$ the whole hierarchical system after such addition. Then, to determine directly the maximum increase in reliability, it will be necessary to calculate the probabilities $P(s_j)$ of fault-free operation for each of the subsystems, then, calculate the probabilities $P(s_j \leftarrow p_{n+1})$ of fault-free operation for each of k subsystems after the addition of another processor, next, on a subsystem level perform probability calculation for resulted hierarchical system $P(S \leftarrow s_j \leftarrow p_{n+1})$, afterward, choose the maximum of resulted values of increase

$$\Delta P(S \leftarrow s_j \leftarrow p_{n+1}) = \\ = P(S \leftarrow s_j \leftarrow p_{n+1}) - P(S).$$

Let us consider a method for determining the maximum increase in reliability, which will reduce the required number of statistical experiments. After decomposing the subsystem $s_j \leftarrow p_{n+1}$, by the added processor, we can get the probability of fault-free operation as:

$$P(s_j \leftarrow p_{n+1}) = p_{n+1} \cdot P(s_j \leftarrow p_{n+1}|_{x_{n+1}=1}) + \\ +(1 - p_{n+1}) \cdot P(S \leftarrow s_j \leftarrow p_{n+1}|_{x_{n+1}=0}).$$

Considering, that if the extra processor fails, subsystems behavior $s_j \leftarrow p_{n+1}$ will be similar to the behavior of the original subsystem s_j, we can assume that:

$$P(s_j \leftarrow p_{n+1}|_{x_{n+1}=0}) \equiv P(s_j).$$

We obtain the following relationship for the increase in subsystem reliability $s_j \leftarrow p_{n+1}$:

$$\Delta P(s_j \leftarrow p_{n+1}) = \\ = p_{n+1} \cdot \left(P(s_j \leftarrow p_{n+1}|_{x_{n+1}=1}) - P(s_j)\right).$$

Similarly, applying decomposition to a hierarchical system $S \leftarrow s_j \leftarrow p_{n+1}$ by the subsystem $s_j \leftarrow p_{n+1}$, we can write down the probability of failure-free operation of a hierarchical system after adding a processor in this form:

$$P(S \leftarrow s_j \leftarrow p_{n+1}) = \\ = P(s_j \leftarrow p_{n+1}) \cdot P\left(S \leftarrow s_j \leftarrow p_{n+1}|_{y_j=1}\right) + \\ + \left(1 - P(s_j \leftarrow p_{n+1})\right) \cdot P\left(S \leftarrow s_j \leftarrow p_{n+1}|_{y_j=0}\right),$$

where $y_j = 1$ is an event, that indicates that $s_j \leftarrow p_{n+1}$ is operable, and $y_j = 0$ is an event of its failure.

Similarly, for the original hierarchical system S we have:

$$P(S) = P(s_j) \cdot P\left(S|_{y_j=1}\right) + \\ + \left(1 - P(s_j)\right) \cdot P\left(S|_{y_j=0}\right).$$

Then the magnitude of reliability increase of the system $S \leftarrow s_j \leftarrow p_{n+1}$ can be presented, as

$$\Delta P(S \leftarrow s_j \leftarrow p_{n+1}) = \\ = P(S \leftarrow s_j \leftarrow p_{n+1}) - P(S) = \\ = P(s_j \leftarrow p_{n+1}) \cdot P\left(S \leftarrow s_j \leftarrow p_{n+1}|_{y_j=1}\right) + \\ + \left(1 - P(s_j \leftarrow p_{n+1})\right) \cdot P\left(S \leftarrow s_j \leftarrow p_{n+1}|_{y_j=0}\right) - \\ - P(s_j) \cdot P\left(S|_{y_j=1}\right) - \left(1 - P(s_j)\right) \cdot P\left(S|_{y_j=0}\right) = \\ = \Delta P(s_j \leftarrow p_{n+1}) \cdot \left(P\left(S|_{y_j=1}\right) - P\left(S|_{y_j=0}\right)\right).$$

Thus, we get:

$$\Delta P(S \leftarrow s_j \leftarrow p_{n+1}) = \\ = p_{n+1} \cdot \left(P(s_j \leftarrow p_{n+1}|_{x_{n+1}=1}) - P(s_j)\right) \times \\ \times \left(P\left(S|_{y_j=1}\right) - P\left(S|_{y_j=0}\right)\right).$$

The resulting relation requires performing statistical experiments for each subsystem to determine the values $P(s_j)$. It is also necessary to perform statistical experiments for the upper-level system to determine the values $P\left(S|_{y_j=1}\right)$ and $P\left(S|_{y_j=0}\right)$, which is commensurate with the definition $P(S \leftarrow s_j \leftarrow p_{n+1})$. However, its obvious advantage is that to define values $P(s_j \leftarrow p_{n+1}|_{x_{n+1}=1})$ it is needed twice as little tests than for values $P(s_j \leftarrow p_{n+1})$, because the variable x_{n+1} is a fixed value.

As well as in previous situations, we can define the expression for the evaluation of p_{n+1}, which is enough for reaching the needed increase $\Delta P_r(S \leftarrow s_j \leftarrow p_{n+1})$ in fault-free system operation probability:

$$p_{n+1} \geq \left(P(s_j \leftarrow p_{n+1}|_{x_{n+1}=1}) - P(s_j)\right) \times \\ \times \frac{\left(P\left(S|_{y_j=1}\right) - P\left(S|_{y_j=0}\right)\right)}{\Delta P_r(S \leftarrow s_j \leftarrow p_{n+1})}.$$

Let us look an example. Let us assume, that all k subsystems have the same number of processors, and the evaluation of $P(s_j)$ for all subsystems is comparable and can be limited by the number of experiments L_1. Assume that evaluation of values $P(s_j \leftarrow p_{n+1})$ requires $2L_1$ experiments and evaluation of $P(S \leftarrow s_j \leftarrow p_{n+1})$ on the upper level requires L_2 experiments. Then directly determining the maximum increase in the reliability of a hierarchical system will require the following number of experiments:

$$k \cdot (kL_1 + 2kL_1 + L_2).$$

The proposed relationship for the same system will require

$$k \cdot (kL_1 + kL_1 + L_2),$$

experiments, meaning kL_1 experiments less.

EXAMPLES

Let us look at some examples. Note, that simplified systems will be considered here, the calculation of reliability parameters of which may be performed without the use of GL-models.

Example 1. The system consists of $n = 10$ identical processors and is resistant to failure of any $m = 2$ of them. The probability of failure of each processor is $q = 10^{-4}$. It is required to estimate the increase in the probability of failure-free operation of such system in case of adding one more similar processor to it.

The probability of failure-free operation of the processor is $p = 1 - q = 1 - 10^{-4} = 0.9999$.

The probability of failure-free operation of the system under review will be:

$$P(S) = \sum_{i=0}^{m} C_n^i p^{n-i} q^i = 0.9999999998800632.$$

Let us also calculate the value of $P(S \leftarrow p)$, i.e., the probability of failure-free operation of a system consisting of 11 processors and resistant to failure of 3 of them:

$$P(S \leftarrow p) = \sum_{i=0}^{m+1} C_{n+1}^i p^{n+1-i} q^i = \\ = 0.999999999999967.$$

Assuming that q is close to 0, we estimate $\nu \approx \frac{m+2}{q(n+1)} = \frac{2+2}{10^{-4}\cdot(10+1)} = 3636.36$. Also let us calculate the direct value of

$$\nu = \frac{1-P(S)}{1-P(S\leftarrow p)} = \frac{1-0.9999999998800632}{1-0.999999999999967} = 3637.3569.$$

Therefore, the estimated value proposed in the article indeed turns out to be very close to the real value.

Let us also estimate $\Delta P(S)$ – the increase in system reliability as a result of adding an extra processor. To do this, we calculate the value of

$$Q(S) = 1 - P(S) = 1.199368 \cdot 10^{-10}.$$

As it has been shown,

$$\Delta P(S \leftarrow p) = \frac{\nu - 1}{\nu} \cdot Q(S) = 1.199 \cdot 10^{-10}.$$

Another formula proposed in the article can also be used:

$$\Delta P(S \leftarrow p) = C_n^{m+1} p^{n-m} q^{m+1} = \\ = C_{10}^{3} \cdot 0.9999^8 \cdot (10^{-4})^3 = 1.199 \cdot 10^{-10}.$$

On the other side, $\Delta P(S) = P(S \leftarrow p) - P(S)$, i.e.,

$$\Delta P(S \leftarrow p) = P(S \leftarrow p) - P(S) = 1.199 \cdot 10^{-10}.$$

As we can see, the value of reliability gain performed by the methods proposed in this article does coincide with the value calculated directly.

Example 2. The system consists of $n = 9$ identical processors and is resistant to failure of any $m = 2$ of them. The probabilities of failure of each processor are $q_1 = 1 \cdot 10^{-4}$, $q_2 = 2 \cdot 10^{-4}$, $q_3 = 3 \cdot 10^{-4}$, $q_4 = 4 \cdot 10^{-4}$, $q_5 = 5 \cdot 10^{-4}$, $q_6 = 6 \cdot 10^{-4}$, $q_7 = 7 \cdot 10^{-4}$, $q_8 = 8 \cdot 10^{-4}$ и $q_9 = 9 \cdot 10^{-4}$, and probabilities of a failure-free work are accordingly $p_1 = 0.9999$, $p_2 = 0.9998$, $p_3 = 0.9997$, $p_4 = 0.9996$, $p_5 = 0.9995$, $p_6 = 0.9994$, $p_7 = 0.9993$, $p_8 = 0.9992$ and $p_9 = 0.9991$.

It is required to estimate the increase in the probability of failure-free operation of such a system in case of adding to it one more processor with some probability of failure-free operation p.

The probability of failure-free operation of such a system can be calculated in one of the known ways, in particular, here and in the following examples we used a method based on statistical experiments with a GL-model of the system behavior in the failure flow [16]. We will not give details of these calculations and will only mention the already obtained results $P(S) = 0.9999999905689658$.

Let us consider several variants of extra processors with probabilities of failure-free operation $p_{10}^{(1)} = 0.999$, $p_{10}^{(2)} = 0.9995$, $p_{10}^{(3)} = 0.9999$. In the same way we can calculate probabilities of failure-free operation of the system, extended by the corresponding processors (considering that such a system will be already 3-fault-tolerant):

$$P(S \leftarrow p_{10}^{(1)}) = 0.9999999999842588;$$
$$P(S \leftarrow p_{10}^{(2)}) = 0.9999999999889713;$$
$$P(S \leftarrow p_{10}^{(3)}) = 0.999999999992741.$$

Therefore,

$$\Delta P(S \leftarrow p_{10}^{(1)}) = P(S \leftarrow p_{10}^{(1)}) - P(S) = 9.4153 \cdot 10^{-9};$$
$$\Delta P(S \leftarrow p_{10}^{(2)}) = P(S \leftarrow p_{10}^{(2)}) - P(S) = 9.42 \cdot 10^{-9};$$
$$\Delta P(S \leftarrow p_{10}^{(3)}) = P(S \leftarrow p_{10}^{(3)}) - P(S) = 9.42378 \times \\ \times 10^{-9}.$$

Note, that the process of direct calculation of the values of $P(S \leftarrow p_{10}^{(1)})$, $P(S \leftarrow p_{10}^{(2)})$ and $P(S \leftarrow p_{10}^{(3)})$ is quite complicated, so we will use the method proposed in the article. To do this, we calculate the value of $P(S^*)$ – the probability that exactly 3 out of 10 processors in the system will fail: $P(S^*) = 9.42472 \cdot 10^{-9}$. Now we can easily calculate the reliability gain values:

$$\Delta P(S \leftarrow p_{10}^{(1)}) = p_{10}^{(1)} \cdot P(S^*) = 0.999 \cdot 9.42472 \times \\ \times 10^{-9} = 9.4153 \cdot 10^{-9},$$
$$\Delta P(S \leftarrow p_{10}^{(2)}) = p_{10}^{(2)} \cdot P(S^*) = 0.9995 \cdot 9.42472 \times \\ \times 10^{-9} = 9.42 \cdot 10^{-9},$$
$$\Delta P(S \leftarrow p_{10}^{(3)}) = p_{10}^{(3)} \cdot P(S^*) = 0.9999 \cdot 9.42472 \times \\ \times 10^{-9} = 9.42378 \cdot 10^{-9}.$$

As we can see, the results obtained by the proposed method match the directly calculated values.

Example 3. Consider a non-basic system consisting of 9 processors and resistant to failures of any two of them, and if the 1st and 2nd and/or 3rd and 4th are operational, then to any three of them. The probabilities of failure-free operation of the system processors are assumed to be the same as in *Example 2*.

It is required to estimate the increase in the probability of failure-free operation of such a system in case of adding to it one more processor with some probability of failure-free operation p. It is assumed that the new system will also be non-basic and resistant to three failures, and if 1st and 2nd and/or 3rd and 4th processors are operational, then to four failures (note that the behavior of the modified system, generally speaking, could be different, and this does not fundamentally affect the possibility of applying the proposed approach).

The probability of failure-free operation of the system described above $P(S) = 0.9999999992112518$. As in the previous case, let us consider several variants of extra processors with the probability of failure-free operation $p_{10}^{(1)} = 0.999$, $p_{10}^{(2)} = 0.9995$, $p_{10}^{(3)} = 0.9999$ and calculate the probabilities of failure-free operation of the system extended by the corresponding processor:

$$P(S \leftarrow p_{10}^{(1)}) = 0.999999999998017;$$
$$P(S \leftarrow p_{10}^{(2)}) = 0.9999999999984108;$$
$$P(S \leftarrow p_{10}^{(3)}) = 0.9999999999987258.$$

Therefore,

$$\Delta P(S \leftarrow p_{10}^{(1)}) = P(S \leftarrow p_{10}^{(1)}) - P(S) = 7.86765 \times \times 10^{-10};$$
$$\Delta P(S \leftarrow p_{10}^{(2)}) = P(S \leftarrow p_{10}^{(2)}) - P(S) = 7.87159 \times \times 10^{-10};$$
$$\Delta P(S \leftarrow p_{10}^{(3)}) = P(S \leftarrow p_{10}^{(3)}) - P(S) = 7.87474 \times \times 10^{-10}.$$

Note that the process of direct calculation of the values of $P(S \leftarrow p_{10}^{(1)})$, $P(S \leftarrow p_{10}^{(2)})$ and $P(S \leftarrow p_{10}^{(3)})$ can be even more complicated than in the previous example so let us use the method proposed in this article.

Let us calculate the probability of failure-free operation of the system S', which is obtained from the original one by adding an infinitely reliable processor, i.e. $p'_{10} = 1$:

$$P(S') = P(S \leftarrow p'_{10}) = 0.9999999999988046.$$

Hence,

$$\Delta P(S') = P(S') - P(S) = 7.875528 \cdot 10^{-10}.$$

Then, as it was shown earlier,

$$\Delta P(S \leftarrow p_{10}^{(1)}) = p_{10}^{(1)} \cdot \Delta P(S') = 0.999 \cdot 7.875528 \times \times 10^{-10} = 7.86765 \cdot 10^{-10};$$
$$\Delta P(S \leftarrow p_{10}^{(2)}) = p_{10}^{(2)} \cdot \Delta P(S') = 0.9995 \times \times 7.875528 \cdot 10^{-10} = 7.87159 \cdot 10^{-10};$$
$$\Delta P(S \leftarrow p_{10}^{(3)}) = p_{10}^{(3)} \cdot \Delta P(S') = 0.9999 \times \times 7.875528 \cdot 10^{-10} = 7.87474 \cdot 10^{-10}.$$

As we can see, in this case, the results obtained by the method proposed in the article match the directly calculated values too.

Example 4. Let us consider the most complicated example. Let the system consist of 21 processors distributed among four subsystems s_1, s_2, s_3 and s_4. Moreover, subsystem s_1 contains processors 1-5, s_2 – processors 6-10, s_3 – processors 11-15 and s_4 – processors 16-21. The system is resistant to failure of any one of the subsystems, and if subsystem s_4 is operational, then to failure of any two subsystems. Subsystem s_1 is non-basic and resistant to failure of one or, if the 2nd or 3rd processor is functional, of two of its processors. Subsystem s_2 is non-basic and resistant to failure of one, and if the 6th processor is functional, of two of its processors. Subsystem s_3 is non-basic and resistant to failure of one processor only if the 11th and/or at the same time the 12th and 13th processors are functional. Subsystem s_4 is non-basic and resistant to failure of one, and if the 16th or 21st processor is functional, of two its processors.

Probabilities of failure-free operation of processors:

$$p_1 = 0.9999,\ p_2 = 0.9998,\ p_3 = 0.9997,\ p_4 = 0.9996,$$
$$p_5 = 0.9995,\ p_6 = 0.9994,\ p_7 = 0.9993,\ p_8 = 0.9992$$
$$p_9 = 0.9991,\ p_{10} = 0.999,\ p_{11} = 0.9989,\ p_{12} = 0.9988,$$
$$p_{13} = 0.9987,\ p_{14} = 0.9986,\ p_{15} = 0.9985,$$
$$p_{16} = 0.9984,\ p_{17} = 0.9983,\ p_{18} = 0.9982,$$
$$p_{19} = 0.9981,\ p_{20} = 0.998,\ p_{21} = 0.997.$$

It is assumed to add a processor (number 22) with some failure-free operation probability p to subsystem s_3. This subsystem is expected to become resistant to one, and, if the 11th and/or at the same time the 12th and 13th and/or at the same time the 12th and 22nd processors are functional, to two failures. It is required to estimate the increase in the probability of system failure-free operation resulting from such a modification.

The probability of failure-free operation of the system described above is $P(S) = 0.9999999999976966$. As in the previous case, consider several variants of extra processors with a probability of failure-free operation $p_{22}^{(1)} = 0.999$, $p_{22}^{(2)} = 0.9995$, $p_{22}^{(3)} = 0.9999$ and calculate the probability of failure-free operation of the system extended by the corresponding processor

$$P(S \leftarrow s_3 \leftarrow p_{22}^{(1)}) = 0.9999999999995793;$$
$$P(S \leftarrow s_3 \leftarrow p_{22}^{(2)}) = 0.9999999999995803;$$
$$P(S \leftarrow s_3 \leftarrow p_{22}^{(3)}) = 0.999999999999581.$$

Consequently, the reliability gain in each case is equal to

$$\Delta P(S \leftarrow s_3 \leftarrow p_{22}^{(1)}) = P(S \leftarrow s_3 \leftarrow p_{22}^{(1)}) - P(S) = = 1.8827 \cdot 10^{-12};$$
$$\Delta P(S \leftarrow s_3 \leftarrow p_{22}^{(2)}) = P(S \leftarrow s_3 \leftarrow p_{22}^{(2)}) - P(S) = = 1.8837 \cdot 10^{-12};$$
$$\Delta P(S \leftarrow s_3 \leftarrow p_{22}^{(3)}) = P(S \leftarrow s_3 \leftarrow p_{22}^{(3)}) - P(S) = = 1.8844 \cdot 10^{-12}.$$

Now let us calculate the reliability gain of the system by the method proposed in the article. Recall that it can be calculated according to the formula

$$\Delta P(S \leftarrow s_3 \leftarrow p_{22}) =$$
$$= p_{22} \cdot \left(P\left(s_3 \leftarrow p_{22}|_{x_{22}=1}\right) - P(s_3)\right) \times$$
$$\times \left(P\left(S|_{y_3=1}\right) - P\left(S|_{y_3=0}\right)\right).$$

Then let us calculate all necessary values of probabilities:

$$P(s_3) = 0.9999831935079323;$$
$$P\left(s_3 \leftarrow p_{22}|_{x_{22}=1}\right) = 0.9999986638033157;$$
$$P\left(s_3 \leftarrow p_{22}|_{x_{22}=1}\right) - P(s_3) = 1.547 \cdot 10^{-5};$$
$$P\left(S|_{y_3=0}\right) = 0.9999998781741908;$$
$$P\left(S|_{y_3=1}\right) = 0.9999999999997441;$$
$$P\left(S|_{y_3=1}\right) - P\left(S|_{y_3=0}\right) = 1.21826 \cdot 10^{-7}.$$

Also let us denote

$$P' = \left(P\left(s_3 \leftarrow p_{22}|_{x_{22}=1}\right) - P(s_3)\right) \times$$
$$\times \left(P\left(S|_{y_3=1}\right) - P\left(S|_{y_3=0}\right)\right) = 1.8846 \cdot 10^{-12}.$$

Now we can calculate the reliability gain for each case:

$$\Delta P(S \leftarrow s_3 \leftarrow p_{22}^{(1)}) = p_{22}^{(1)} \cdot P' = 1.8827 \cdot 10^{-12};$$
$$\Delta P(S \leftarrow s_3 \leftarrow p_{22}^{(2)}) = p_{22}^{(2)} \cdot P' = 1.8837 \cdot 10^{-12};$$
$$\Delta P(S \leftarrow s_3 \leftarrow p_{22}^{(3)}) = p_{22}^{(3)} \cdot P' = 1.8844 \cdot 10^{-12}.$$

As we can see, the values calculated in this way match the values calculated directly.

It is readily apparent that for each of the presented examples (bearing in mind that each involved the consideration of three candidate processors), the overall complexity of the incremental reliability assessment procedure was reduced by approximately threefold due to the absence of the need for direct reliability calculation for each variant of the modified system.

CONCLUSIONS

The work solves the problem of assessing the magnitude increase of failure-free operation probability for a fault-tolerant multiprocessor control system after adding an extra processor. Various types of systems are researched: basic with the same processors, basic with different processors, and non-basic, including hierarchical systems consisting of several subsystems.

The results obtained in the work can be used by developers of fault-tolerant multi-processor systems both to assess the possibilities for increasing the reliability of the developed system and to select the optimal configuration of the modified system and select the appropriate element base (for example, putting forward requirements for the reliability parameters of the added processor).

It should be noted that in each case, the increase in reliability is directly proportional to the probability of fault-free operation of the added processor. This allows you to perform the most complex calculations for the selected configuration of the modified system once, and then just substitute the values of the probability of failure-free operation for the considered variants of the added processor into a basic expression. In addition, it turned out to be possible to estimate the minimum sufficient value of the probability of failure-free operation of such a processor to achieve needed increase in reliability (or the impossibility of achieving it by adding just one processor).

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Conflicts of Interest: the authors declare no conflict of interest

Received 12.01.2024
Received after revision 14.03.2024
Accepted 21.03.2024

DOI: https://doi.org/10.15276/aait.07.2024.7
УДК 004.05

Про оцінку приросту надійності відмовостійких багатопроцесорних систем

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АНОТАЦІЯ

Робота присвячена задачі оцінки приросту надійності відмовостійкої багатопроцесорної системи в результаті додавання до неї додаткового процесора. Передбачається, що поведінка модифікованої системи потоці відмов у разі відмови додаткового процесора не відрізняється від поведінки вихідної системи. У статті розглядаються як системи виду *k*-з-*n*, так і складніші, зокрема, ієрархічні. Важливою особливістю запропонованого підходу є те, що він передбачає попередній розрахунок деяких допоміжних значень, які не залежать від параметрів надійності процесора, що додається. Далі оцінка приросту надійності виконується шляхом підстановки значень цих параметрів у прості вирази, що дозволяє спростити вибір оптимального процесора з множини доступних, достатнього для досягнення необхідного рівня надійності системи, або переконатися у неможливості цього. Запропонований підхід сумісний з будь-якими методами розрахунку параметрів надійності відмовостійких багатопроцесорних систем, але особливо актуальний для методів, що базуються на проведенні статистичних експериментів з моделями поведінки системи в потоці відмов, зокрема такими, як GL-моделі, внаслідок суттєвої обчислювальної складності таких розрахунків. Крім того, для найбільш простих випадків, що розглядаються, систем виду *k*-з-*n* з однаковими процесорами, запропоновано простий вираз для приблизної оцінки співвідношення ймовірностей виходу з ладу вихідної і модифікованої системи. Точність такої оцінки виявляється тим вищою, чим вище надійність процесорів системи. Наведено приклади, які на практиці доводять коректність запропонованих підходів. Розрахунок значень параметрів надійності системи, як і допоміжних виразів, був виконаний на основі проведення статистичних експериментів з відповідними GL-моделями.

Ключові слова: відмовостійкі багатопроцесорні системи; підвищення надійності; системи *k*-з-*n;* ієрархічні системи; GL-моделі

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